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PRIMER OF GEOMETRY

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AN EASY INTRODUCTION

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THE PROPOSITIONS OF EUCLID

BY

FRANCIS CUTHBERTSON, M.A., LL.D.

LATE FELLOW OF CORPUS CHRISTI COLL., CAMBRIDGE HEAD MATHEMATICAL MASTER OF THE CITY OF LONDON SCHOOL

WILLIAM BLACKWOOD AND SONS EDINBURGH AND LONDON MDCCCLXXVI

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PREFACE.

WHILST many of the Propositions of Euclid are very easy, there are others equally hard for the student to understand; moreover, they are not graduated according to their difficulty. Thus the Fifth Proposition of his First Book is invariably found to be a greater stumbling-block than most of those which follow. Now the earliest time at which the study of the Elements can be commenced is necessarily fixed by the difficult parts. Accordingly, inasmuch as those whose capacity is considerably below the limit thus indicated could master the easy portions, it follows that, by means of a course of Geometry embracing these, the subject might be introduced much sooner than it usually is.

As the beginner is always more interested in Problems than Theorems, one half of this work will be found to consist of Problems. The solu-

• Preface.

tions to these are not only simple, but also practical. This is important, as the constructions generally given in theoretical works on Geometry are not employed in practice, whilst those in use are selected from treatises on Practical Geometry, and have no proofs attached to them. The Problems are mainly those of the first four Books of Euclid. The demonstrations of these, as also of the Theorems, assume no more than Euclid does in the corresponding Propositions, and may accordingly be substituted for them in examinations. This is therefore not merely a course preparatory to a more extended study of Geometry, but, in the event of its being followed by Euclid as a text-book, a great deal of the ground of his first four Books will then have been already covered by it, and in such a manner as to render the completion a matter of no great labour.

CITY OF LONDON SCHOOL, October 1875.

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PRELIMINARY NOTICES.

THE following axioms will be assumed :---

Things which are equal to the same thing are equal to one another.

The whole is equal to the sum of its parts.

If equals be added to equals the wholes are equal.

If equals be taken from equals the remainders are equal.

Things which are double of equal things are equal to one another.

Things which are halves of equal things are equal to one another.

After the first few Propositions the following symbols will be introduced :---

7	•	•	•	angle.
Δ		•		triangle.
		•		parallelogram.
=			•	equal to.
··.	•		•	therefore.

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PRIMER OF GEOMETRY.

BOOK I.

INTRODUCTION.

Definitions &c.

A straight line is one which lies evenly between its extremities.

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EUCLID'S AXIOM. Two straight lines cannot enclose a space.

If two straight lines be drawn from a point, they will form an **angle**. Those two lines are called the **arms**, and that point is called the **vertex** of the angle.

The size of an angle does not depend on the length of its arms, but only on their direction.

If one angle can be placed on another so that the vertex of one may coincide with the vertex of the other, and the arms of one lie along the arms of the other, the angles are said to be equal to one another.

DEFINITIONS &c.

A triangle is a plane figure bounded by three straight lines, called the sides of the triangle.

The three angles formed by the sides are called the angles of the triangle.

If one triangle can be placed on another so that the boundary of one may coincide with the boundary of the other, then the triangles are equal to one another, the sides of the one are equal to the sides of the other, and the angles of the one to the angles of the other. The triangles are then said to be equal in all respects.

A circle is a plane figure bounded by one line, called the circumference, and is such that all straight lines drawn from a certain point within the figure to the circumference are equal.

That point is called the centre, and those straight lines are called radii.

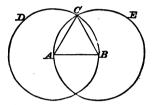
STRAIGHT LINES, ANGLES, AND TRIANGLES.

DEFINITION.

An equilateral triangle is one which has all its sides equal.

PROPOSITION I.

To describe an equilateral triangle upon a given straight line.



Let AB be the given straight line.

It is required to describe an equilateral triangle upon AB.

With centre A, and radius AB, describe the circle BCD; and with centre B, and radius BA, describe the circle ACE.

From the point C, in which these circles cut one another, draw the straight lines CA, CB.

Then ABC shall be an equilateral triangle,

Because A is the centre of the circle BCD, therefore AC is equal to AB;

and because B is the centre of the circle ACE, therefore BC is equal to AB.

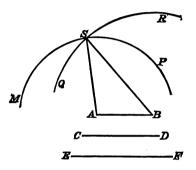
Thus AC and BC are each of them equal to AB.

Therefore AC and BC are equal to one another.

Therefore AC, BC, and AB are equal to one another. Therefore the triangle ABC is equilateral.

Wherefore an equilateral triangle has been described upon AB. WHICH WAS TO BE DONE. PROPOSITION II.

To describe a triangle having its sides equal to three given straight lines.



Let AB, CD, and EF be the three given straight lines. It is required to describe a triangle having its sides equal to AB, CD, and EF respectively.

With centre A, and radius equal to CD, describe the circle MP;

and with centre B, and radius equal to EF, describe the circle QR.

If these circles intersect in S, join AS and BS. Then shall ABS be the triangle required.

For AS is a radius of the circle MP, and is therefore equal to CD;

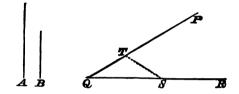
also BS is a radius of the circle QR, and is therefore equal to EF.

Wherefore a triangle ABS has been described having its sides AB, AS, BS respectively equal to the given straight lines AB, CD, EF. QUOD ERAT FACIENDUM.

Note.—If the circles do not intersect, the problem cannot be done.

PROPOSITION III.

To describe a triangle having two of its sides equal to two given straight lines, and the angle contained by those sides equal to a given angle.



Let A and B be the given straight lines, and PQR the given angle.

It is required to describe a triangle having two of its sides equal to A and B, and the angle contained by those sides equal to the angle PQR.

From QR cut off QS, equal to A; and from QP cut off QT, equal to B. Join ST. Then shall the triangle QST be the one required.

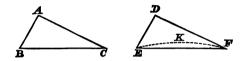
For the triangle QST has two of its sides, QS, QT, equal to the two given straight lines A and B, and they contain the angle TQS, which is the same as the given angle PQR.

Wherefore a triangle has been described having two of its sides &c. Q.E.E.

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PROPOSITION IV.

If two sides and the included angle of one triangle be respectively equal to two sides and the included angle of another triangle, the triangles shall be equal in all respects.



In the triangles ABC, DEF,

let the two sides AB, AC, and the included angle BAC, be respectively equal to the two sides DE, DF, and the included angle EDF.

Then shall the triangles ABC, DEF be equal in all respects.

For if the triangle ABC were applied to the triangle DEF so that the point A fell on the point D, and the \cdot straight line AB along DE, then would AC fall along DF, because the angle BAC is equal to the angle EDF;

also B would fall on E, because AB is equal to DE; and C on F, because AC is equal to DF.

Therefore the straight line BC would coincide with EF; for if it fell otherwise, as EKF, then two straight lines would enclose a space, which is impossible.

Therefore BC would coincide with EF, and is therefore equal to it; and the triangle ABC would coincide with the triangle DEF, and is equal to it.

So the angle ABC would coincide with the angle DEF, and is equal to it; and the angle ACB would coincide with the angle DFE, and is equal to it.

Thus the triangle ABC is equal to the triangle DEF in all respects.

Wherefore if two sides &c.

WHICH WAS TO BE PROVED.

DEFINITION.

An isosceles triangle is one which has two of its sides equal.

PROPOSITION V.

The angles at the base of an isosceles triangle are equal to one another.



Let ABC be an isosceles triangle, having the side AB equal to the side AC.

Then shall the angle ABC be equal to the angle ACB.

For if the triangle were taken up, reversed, and then placed so that A fell on its former position,

and AB along the former position of AC; then would AC fall along that of AB;

also B would fall on that of C, because AB is equal to AC, and C on that of B for the same reason.

Therefore BC would fall on its former position.

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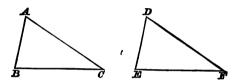
Thus AB, BC would fall on the former positions of AC, CB;

Therefore the angle ABC is equal to the angle ACB. Wherefore the angles at the base &c.

QUOD ERAT DEMONSTRANDUM.

PROPOSITION VI.

If two angles and the adjacent side of one triangle be respectively equal to two angles and the adjacent side of another triangle, then the triangles shall be equal in all respects.



In the triangles ABC, DEF, let the two angles ABC, ACB, and the adjacent side BC, be respectively equal to the two angles DEF, DFE, and the adjacent side EF.

Then shall the triangles ABC DEF be equal in all respects.

For if the triangle ABC were applied to the triangle DEF, so that BC fell upon EF,

then BA would fall along ED, since the angle ABC is equal to the angle DEF;

and CA would fall along FD, since the angle ACB is equal to the angle DFE.

Therefore the point A would fall on the point D, and the triangle ABC would coincide with the triangle DEF.

Therefore the triangles ABC, DEF are equal in all respects.

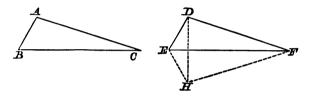
Wherefore if two angles and the adjacent side &c.

Q.E.D.

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PROPOSITION VII.

If the three sides of one triangle be respectively equal to the three sides of another, the triangles shall be equal in all respects.



Let the sides AB, BC, CA of the $\triangle ABC$ be respectively equal to the sides DE, EF, FD of the $\triangle DEF$.

Then shall the triangles ABC, DEF be equal in all respects.

For if the $\triangle ABC$ were taken up, reversed, and placed so that BC fell on EF,

and A on the opposite side of EF to that on which D is, as at H,

and HD joined;

then would $\angle EHD$ be equal to $\angle EDH$, because EH is equal to ED; (I. 5)

and \angle FHD be equal to \angle FDH, because EH is equal to ED: (I r)

because FH is equal to FD; (I. 5) and therefore $\angle EHF$ equal to $\angle EDF$,

that is, $\angle BAC$ is equal to $\angle EDF$,

also the sides AB, AC are respectively equal to the sides DE, DF.

Therefore the $\triangle ABC$ is equal to the $\triangle DEF$ in all respects. (I. 4)

Wherefore if three sides of one triangle &c. Q.E.D.

Note.—For convenience, the side BC of $\triangle ABC$, which is not less than either of the others, has been applied to the equal side of $\triangle DEF$.

PROPOSITION VIII. To bisect a given angle.



Let DAE be the given angle. It is required to bisect it.

With centre A describe a circle cutting AD, AiB and C.

With centres B and C describe circles having erradii, intersecting in F.

Join AF. Then $\angle DAE$ shall be bisected by AF.

Join BF and CF.

Because in the $\Delta s ABF, ACF$,

the side AF is common to both;

also BA is equal to CA, because they are radii of same circle,

and BF is equal to CF, because they are radi equal circles;

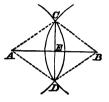
therefore the Δ s *ABF*, *ACF* are equal in all spects, (1)

and thus $\angle BAF$ is equal to the $\angle CAF$.

Wherefore $\angle DAE$ has been bisected.

PROPOSITION IX.

To bisect a given straight line.



Let AB be the given straight line. It is required to bisect AB.

With centres A and B describe two circles having equal radii intersecting in C and D.

Join CD cutting AB in E.

Then AB shall be bisected in E.

Join AC, CB, BD, DA.

Now because in the Δ s ACD, BCD the sides AC, CD, DA are respectively equal to BC, CD, DB,

therefore the $\Delta s ACD$, BCD are equal in all respects,

and thus the $\angle ACD$ is equal to the $\angle BCD$. (I. 7) Hence in the $\triangle s ACE$, BCE, the two sides AC, CE and the included angle ACE are respectively equal to the two sides BC, CE and the included angle BCE.

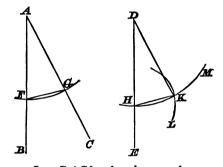
Therefore the Δs ACE, BCE are equal in all respects, and therefore AE is equal to BE. (I. 4)

Wherefore the given straight line AB has been bisected.

Q.E.E.

PROPOSITION X.

At a point in a given straight line to make an angle equal to a given angle.



Let BAC be the given angle. It is required to make at the point D in the straight line DE an angle equal the given angle BAC.

With centre A describe a circle cutting AB and ACin F and G; and with centre D describe a circle HKMwith equal radius cutting DE in H.

Join FG; and with centre H and radius equal to FG describe a circle cutting the circle HKM in K.

Join *DK*.

Then EDK shall be the angle required.

Join HK.

Then because in the Δs HDK, FAG the sides HD, DK, and KH are respectively equal to FA, AG, and GF;

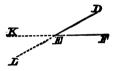
therefore $\angle HDK$ is equal to $\angle FAG$. (I. 7)

Wherefore at the point D in the straight line DE an angle EDK has been made equal to the given angle BAC. Q.E.F.

VERTICAL ANGLES.

PROPOSITION XI.

If two straight lines cut one another, the vertical (or opposite) angles shall be equal to one another.



Let the two straight lines *DEL*, *FEK* cut one another.

Then shall $\angle DEF$ be equal to the vertical $\angle KEL$.

For if the figure were taken up, reversed, and placed so that each of the arms of $\angle DEK$ might fall along the former position of the other arm; that is, so that

ED might fall along the former position of EK, and EK along that of ED;

then would EK produced, or EF, fall along that of ED produced, or EL.

Thus EF and ED would fall along the former positions of EL and EK.

Therefore $\angle DEF$ is equal to $\angle KEL$.

Wherefore if two straight lines &c.

Q.K.D.

DEFINITION.

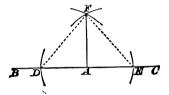
When one straight line, standing on another straight line, makes the adjacent angles equal to one another, each of them is called **a right angle**;

and the straight line which stands upon the other is said to be **perpendicular** to it.

EUCLID'S AXIOM.—All right angles are equal to one another.

PROPOSITION XII.

To draw a perpendicular to a given straight line from a given point in the same.



Let BC be the given straight line, and A the given point in it. It is required to draw from A a perpendicular to BC.

From AB, AC cut off equal parts AD, AE.

With D and E as centres, describe circles with equal radii intersecting in F.

Join AF. Then shall AF be perpendicular to BC.

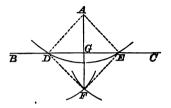
Join DF and EF.

Because the side AF is common to the Δ s DAF, EAF, and AD is equal to AE, and also DF is equal to EF; therefore Δ s DAF, EAF are equal in all respects, (I. 7) and thus $\angle DAF$ is equal to $\angle EAF$.

Therefore AF is perpendicular to BC .	(Def.)
Wherefore from a given point &c.	Q.E.F.

PROPOSITION XIII.

To draw a perpendicular to a given straight line from a given point without it.



Let A be the given point without the given straight line BC.

It is required to draw from A a perpendicular to BC.

With centre A describe a circle cutting BC at D and E.

With centres D and E describe circles with equal radii intersecting in F.

Join AF, cutting BC in G.

Then shall AG be perpendicular to BC.

Join AD, AE, FD, and FE.

Now because in the Δs *DAF*, *EAF* the sides *DA*, *AF*, and *FD* are respectively equal to *EA*, *AF*, and *FE*; $\therefore \angle DAF$ is equal to $\angle EAF$. (I. 7)

Again, because in the $\Delta s DAG$, EAG the sides DA, AG and the included angle DAG are respectively equal to EA, AG and the included angle EAG;

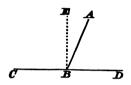
 $\therefore \perp DGA$ is equal to $\perp EGA$; (I. 4)

 $\therefore AG$ is perpendicular to *BC*. (Def.)

Wherefore from a given point &c. Q.E.F.

PROPOSITION XIV.

The angles which one straight line makes with another upon one side of it are either two right angles or are together equal to two right angles.



Let the straight line AB make with CD upon one side of it the $\angle s ABC$, ABD.

These shall either be two right angles or shall together be equal to two right angles.

If the $\angle ABC$ is equal to the $\angle ABD$;

then each of them is a right angle. (Def.) But if not, through B draw BE perpendicular to CD; (I. 12)

then the $\angle ABC$ is equal to the $\angle SCBE$ and EBA together;

therefore the $\angle s \ ABC$ and ABD are together equal to the $\angle s \ CBE$, EBA, and ABD,

of which $\angle CBE$ is a right angle,

and \angle s *EBA* and *ABD* together make up a right angle.

Therefore the \angle s *ABC* and *ABD* are together equal to two right angles.

Wherefore the angles which one straight line &c.

Q. E. D.

COROLLARY.—If two angles be equal to one another, then the adjacent angles, formed by producing an arm of each through the vertex, will be equal to one another.

PARALLELS.

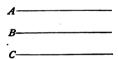
DEFINITION.

Straight lines, lying in the same plane and which will never meet, though produced ever so far both ways, are called **parallel**.

PLAYFAIR'S AXIOM.—Two straight lines which cut one another cannot be parallel to the same straight line.

• PROPOSITION XV.

Straight lines which are parallel to the same straight line are parallel to one another.



Let the straight lines A and B be each of them parallel to C.

Then shall A and B be parallel to one another.

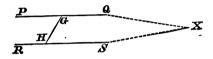
For if not, they will meet if produced, and there will thus be two intersecting straight lines parallel to the same straight line, which is impossible. (Axiom.)

Wherefore straight lines &c.

Q.E.D.

PROPOSITION XVI.

If, in a plane, a straight line, meeting two other straight lines, make the alternate angles equal to one another, these two straight lines shall be parallel.



Let the straight line GH, meeting the two straight lines PQ, RS, make the alternate $\angle s$ HGP, GHSequal to one another, and therefore also the alternate $\angle s$ HGQ, GHR equal to one another. (I. 14, Cor.) Then shall PQ, RS be parallel.

For if not, PQ, RS when produced will meet either towards Q and S or towards R and P.

Let them, if possible, meet when produced towards Q and S in the point X.

Thus GHX is a Δ , and if it were applied to GHon the opposite side, then HX would fall along GP, because $\angle HGP$ is equal to $\angle GHS$; and GX would fall along HR, because $\angle GHR$ is equal to $\angle HGQ$.

Therefore GP, HR would meet if produced towards P and R, and thus two straight lines would enclose a space, which is impossible.

Therefore PQ, RS will not meet if produced towards Q and S.

Similarly it may be shown that they will not meet if produced towards P and R.

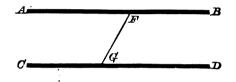
Therefore PQ and RS are parallel.

Wherefore if in a plane a straight line &c. Q.E.D.

Parallels.

PROPOSITION XVII.

To draw a straight line through a given point parallel to a given straight line.



Let F be the given point, and CD the given straight line.

It is required to draw a straight line through F parallel to CD.

Join F with any point G in CD.

At the point F in FG make $\angle GFA$ equal to $\angle FGD$, and produce AF to B. (I. 10)

Then shall AB be parallel to CD.

For since FG, meeting the two straight lines AB and CD, makes the alternate angles GFA, FGD equal to one another,

therefore AB is parallel to CD. (I. 16)

Wherefore a straight line has been drawn &c. Q.E.F.

PROPOSITION XVIII.

If a straight line meet two parallel straight lines, the alternate angles shall be equal to one another.



Let the straight line FG meet the two parallel straight lines AB and CD.

Then shall $\angle GFA$ be equal to the alternate $\angle FGD$.

For, if possible, let them be unequal, and at F in FGmake $\angle GFX$ equal to $\angle FGD$. (I. 10)

Then XF is parallel to CD. (I. 16)

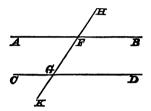
Therefore through F two straight lines have been drawn parallel to CD, which is impossible.

Therefore $\angle GFA$ is not unequal to $\angle FGD$ —that is, it is equal to it.

Wherefore if a straight line &c. Q.E.D.

Parallels.

COROLLARY 1.—If a straight line fall upon two parallel straight lines, the exterior angle shall be equal to the interior and opposite angle upon the same side of the line.



Let HK fall upon the two parallel straight lines AB, CD;

then shall $\angle HFB$ be = $\angle FGD$.

For $\angle AFG$ is = the alternate $\angle FGD$;

and $\angle AFG$ is = the vertical opposite $\angle HFB$; (I. 11) therefore $\angle HFB$ is = $\angle FGD$.

COROLLARY 2.—If a straight line fall upon two parallel straight lines, the two interior angles upon the same side of the line shall together be equal to two right angles.

That is, the \angle s *BFG*, *FGD* shall be together equal to two right angles.

For $\angle AFG$ is = the alternate $\angle FGD$;

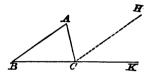
therefore the \angle s *BFG*, *AFG* are together = the \angle s *BFG*, *FGD*;

but the \angle s *BFG*, *AFG* are together = two right angles; (I. 14)

therefore $\angle s$ BFG, FGD are together = two right angles.

PROPOSITION XIX.

If one side of a triangle be produced, the exterior angle shall be equal to the two interior and opposite angles.



Let ABC be a Δ , and let one of its sides BC be produced to K.

Then shall the exterior angle ACK be equal to the two interior and opposite angles BAC, ABC.

Through C draw CH parallel to AB. (I. 17)

Now because CH is parallel to AB, and AC meets them,

... the $\angle ACH$ is = the alternate $\angle BAC$. (I. 18) Again, because CH is parallel to AB, and BK falls upon them;

: the exterior \angle HCK is = the interior and opposite \angle ABC; (I. 18, Cor.)

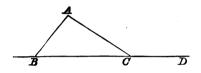
 \therefore the whole $\angle ACK$ is = the two angles BAC, ABC together.

Wherefore if one side of a triangle &c. Q.E.D.

Parallels.

PROPOSITION XX.

The three angles of a triangle are together equal to two right angles.



Let ABC be a Δ .

Then shall its three angles be together equal to two right angles.

Produce BC to D.

Then the exterior angle ACD is = the two $\angle s ABC$, BAC together. (I. 19)

Add to each of these equals the $\angle ACB$;

: the \angle s ACD, ACB are together = the \angle s ABC, BAC, ACB,

i.e. = the three $\angle s$ of $\triangle ABC$.

But the $\angle s \ ACD$, ACB are together = two right angles; (I. 14) \therefore the three $\angle s$ of $\triangle ABC$ are together = two right $\angle s$. Q.E.D.

PARALLELOGRAMS.

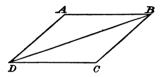
DEFINITIONS.

A parallelogram is a four-sided figure, of which the opposite sides are parallel.

A straight line joining two opposite angles of a parallelogram is called a diagonal.

PROPOSITION XXI.

The opposite sides and angles of a parallelogram are equal, and the diagonal bisects it.



Let ABCD be a \square , and BD a diagonal. Then shall AB be = DC, AD = BC, $\angle BAD = \angle BCD, \angle ABC = \angle ADC,$ and $\triangle ABD = \triangle CDB$.

Because AB is parallel to DC. \therefore the alternate \angle s ABD, BDC are equal to one another, (I. 18)

and because AD is parallel to BC,

 \therefore the alternate $\angle s ADB$, DBC are equal to one (I. 18) another:

 \therefore the whole $\angle ABC$ is = the whole $\angle ADC$.

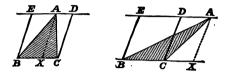
Also, because in the \triangle s *ABD*, *CDB* the \perp s *ABD*, ADB are respectively equal to the \angle s CDB, CBD, and the side DB adjacent to the equal $\angle s$ in each is common;

 \therefore the \triangle s *ABD*, *CDB* are equal in all respects, and AB, AD, and $\angle BAD$ are respectively equal to DC. BC, and \angle BCD. (I. 6) Q.E.D.

Wherefore the opposite sides &c.

PROPOSITION XXII.

If a parallelogram and a triangle be on the same base and between the same parallels, the parallelogram shall be double of the triangle.



Let the \square *EBCD* and the \triangle *ABC* be on the same base *BC*, and between the same parallels.

Then shall \square *EBCD* be double of the \triangle *ABC*.

If the vertex A of the $\triangle ABC$ be at D or E the proposition is evident. (I. 21)

But if not, through A draw AX parallel to EB or DC, meeting the base or base produced in X.

Then EX is a \square and is \therefore double of $\triangle ABX$; (I. 21) also DX is a \square and is \therefore double of $\triangle ACX$; (I. 21) $\therefore \square EC$ is double of $\triangle ABC$.

Wherefore if a parallelogram &c. Q.E.D.

COROLLARY 1.—If there be any number of \Box s on the same base and between the same parallels, they will be equal to one another.

For each of them will be double of any Δ which may be drawn on that base and between those parallels.

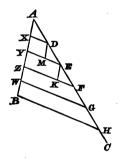
COROLLARY 2.—If there be any number of Δs on the same base and between the same parallels, they will be equal to one another.

For each of them will be half of any \square which may be drawn on that base and between those parallels.

Primer of Geometry.

PROPOSITION XXIII.

To divide a straight line into any given number of equal parts.



Let it be required to divide the straight line AB into five equal parts.

From A draw another straight line AD, and from AD produced cut off DE, EF, FG, GH, each equal to AD.

Join HB, and draw DX, EY, FZ, GW parallel to it; (I. 17)

and cutting AB in X, Y, Z, W.

Then shall AB be divided into five equal parts.

Draw DM, EK parallel to AB.

Then because DM is parallel to AB, and AE falls upon them,

 $\therefore \ \ \ MDE = \ \ \ \ XAD, \qquad (I. 18, Cor. 1)$

b

and because EM is parallel to DX, and AE falls upon them,

... $\angle DEM = \angle ADX.$ (I. 18, Cor. 1) Also DE is = AD, ... the \triangle s DEM, ADX are equal in all respects, (I. 6) and thus DM is equal to AX. But DM = XY, (I. 21) ... XY = AX. Similarly YZ, ZW, and WB may be proved to be equal to AX.

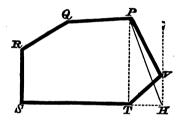
Hence AB is divided into five equal parts.

In like manner may a straight line be divided into any given number of equal parts. Q.E.F.

Primer of Geometry.

PROPOSITION XXIV.

To describe a triangle equal to a given rectilinea. figure.



Let *PQRSTV* be the given rectilineal figure. It is required to describe a triangle equal to it.

Join PT.

Draw through V the straight line VH parallel to PT, meeting ST produced in H. (I. 17)

Join PH.

Then since the Δs *PTH*, *PTV* are upon the same base *PT*,

and between the same parallels VH, PT;

Parallelograms.

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 $\therefore \Delta PTH = \Delta PTV. \qquad (I. 22, Cor. 2)$

Add to each of these equals the figure PQRST;

... the figure PQRSH is = figure PQRSTV.

Similarly a figure may be described = PQRSH having one fewer sides; and so on.

Thus a triangle may be described &c. Q.E.F.

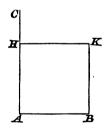
SQUARES.

DEFINITION.

A square is a four-sided figure, having all its sides equal to one another, and all its angles right angles.

PROPOSITION XXV

To describe a square on a given straight line.



Let AB be the given straight line. It is required to describe a square on AB.

Through A draw AC perpendicular to AB, (I. 12) and from AC cut off AH = AB. Through H draw HK parallel to AB, (I. 17) Squares.

and through B draw BK parallel to AH, (I. 17) and meeting HK in K. Then shall ABKH be the square required.

For, by construction, ABKH is a parallelogram.

 $\therefore AB = HK, \text{ and } AH \text{ is } = BK; \quad (I. 21)$

also AH was made = AB;

.:. ABKH has all its sides equal.

Again, since HK is parallel to AB,

but $\angle BAH$ is a right angle, $\therefore \angle AHK$ is a right angle.

 \therefore the \angle s *BKH*, *ABK* opposite to these are right angles; (I. 21)

 \therefore all the angles of *ABKH* are right angles.

Therefore ABKH is a square.

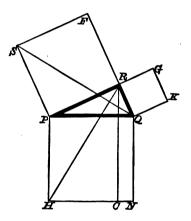
Wherefore a square has been described on AB.

Q.E.F.

COROLLARY.—If one angle of a parallelogram is a right angle, all its angles are right angles.

PROPOSITION XXVI.

In any right-angled triangle, the square described on the side opposite the right angle is equal to the squares described on the sides containing the right angle.



Let PQR be a right-angled triangle.

Then shall the square on PQ, the side opposite the right angle, be equal to the squares on PR and RQ.

Produce PR and QR, then the \angle s at R are right angles.

Complete the squares on PR and QR, and describe the square on PQ. (I. 25) Through R draw RC parallel to PH. Join RH and SQ.

Then the right \angle SPR is = the right \angle QPH;

add to each the $\angle RPQ$;

 \therefore the $\angle SPQ$ is = the $\angle RPH$.

Also the sides SP, PQ are respectively = the sides RP, PH;

 \therefore the \triangle SPQ is = the \triangle RPH. (I. 4)

But the $\square PC$ is double of the $\triangle RPH$, because they are on the same base *PH*, and between the same parallels; (I. 22)

and so likewise SPRF is double of the \triangle SPQ; (I. 22)

 \therefore the $\square PC$ is = SPRF which is the square on PR.

Similarly it may be proved that the $\Box QC$ is = the square on RQ.

 \therefore the whole figure *PHNQ*, which is the square no *PQ*, is = the squares on *PR* and *RQ*.

Wherefore in any right-angled triangle &c. Q.E.D.

BOOK II.

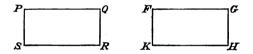
RECTANGLES.

DEFINITION.

A rectangle is a right-angled parallelogram.

PROPOSITION I.

If two adjacent sides of one rectangle be respectively equal to two adjacent sides of another, the rectangles shall be equal to one another.



Let the two adjacent sides PQ, QR of the rectangle PR be respectively equal to the two adjacent sides FG, GH of the rectangle FH.

Then shall the rectangle PR be equal to the rectangle FH.

For let PR be applied to FH, so that PQ may fall on FG,

Rectangles.

then will QR fall along GH, since $\angle PQR = \angle FGH$, and R will fall on H, since QR = GH;

so likewise will RS and SP fall on HK and KF.

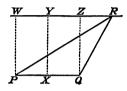
Thus the figure PQRS will coincide with the figure FGHK, and is therefore equal to it.

Wherefore if two adjacent sides &c. Q.E.D.

Note.—A rectangle is said to be contained by any two of its adjacent sides: thus the rectangle PQRS will be spoken of as the "rectangle contained by PQ, QR," which for brevity may be written thus—" the rectangle PQ, QR."

PROPOSITION II.

To describe a rectangle equal to a given triangle.



Let PQR be the given triangle.

It is required to describe a rectangle equal to the \triangle PQR.

Through R draw a straight line RW parallel to PQ. (I. 17)

Bisect PQ in X .	(I. 9)
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- Draw XY perpendicular to PQ, (I. 12)
 - and QZ parallel to XY. (I. 17)

Then XQZY is a rectangle;

also it shall be the one required.

Draw PW parallel to XY. (I. 17)

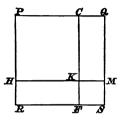
Then rectangle PY = rectangle QY, (II. 1) $\therefore QY$ is half of QW.

But $\triangle PQR$ is half of QW, (I. 22) \therefore rectangle $QY = \triangle PQR$.

Wherefore a rectangle has been described equal to the given triangle. Q.E.F.

PROPOSITION III.

If a straight line be divided into any two parts, the square on the whole line will be equal to the squares on the two parts, together with twice the rectangle contained by the parts.



Let the straight line PQ be divided into any two parts in C.

Then shall the square on PQ be equal to the squares on PC, CQ, together with twice the rectangle contained by PC, CQ.

Draw *PR* perpendicular to *PQ*, and equal to it. (I. 12) Cut off PH = PC, so that *HR* is = CQ.

Draw HM and RS parallel to PQ; (I. 17) also CF and QS parallel to PR. (I. 17)

Then *PRSQ* is a square, (I. 25) and it is equal to the figures *PK*, *KS*, *CM*, *HF* together.

Now PK is the square on PC; (I. 25) KS is the square on KM, and is therefore = the square on CQ, for KM is = CQ; (I. 21)

CM is the rectangle contained by CK, CQ, and is therefore = the rectangle PC, CQ, for CK is = PC;

and HF is also = rectangle \overline{PC} , CQ, for HK is = PC, and HR = CQ.

Therefore the square on PQ is = the squares on PC, CQ, together with twice the rectangle PC, CQ.

Wherefore if a straight line &c. Q.E.D.

PROPOSITION IV.

If a straight line be divided into any two parts, the square on one part will be less than the squares on the whole and on the other part by twice the rectangle contained by the whole and that other part.



Let the straight line AB be divided into any two parts in C.

Then shall the square on AC be less than the squares \cdot on AB and BC by twice the rectangle AB, BC.

Draw AGS perpendicular to AB; (I. 12)

also QCH and RBT parallel to AGS. (I. 17)

Cut off AG = AC, AS = AB, and BR = BC.

Draw GHF, ST, and RQ parallel to AB. (I. 17)

Then AH, AT, and BQ are squares, (I. 25) and we have to show that

Rectangles.

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AH is less than AT and BQ by twice the rectangle AB, BC;

that is, that the figure GTQ is = twice the rectangle AB, BC.

Now the figure GT is = the rectangle AB, BC, for GF is = AB, and GS = BC;

also the figure QF is = the rectangle AB, BC,

for RF is = AB, and QR = BC.

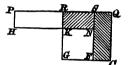
Therefore the figure GTQ is = twice the rectangle AB, BC.

Therefore the square on AC is less than the squares on AB, BC by twice the rectangle AB, BC.

Wherefore if a straight line &c. Q.E.D.

PROPOSITION V.

The difference between the squares on any two straight lines is equal to the rectangle contained by two straight lines, one of which is equal to the sum, and the other equal to the difference of the given lines.



Let RQ, RS be equal to the given straight lines. Then SQ is = the difference between them. Produce QR to P, making RP = QR. Then PS is = the sum of the given lines.

The difference between the squares on RQ and RS shall be equal to the rectangle PS, SQ.

On RQ describe a square RGCQ. (I. 25) Draw PH perpendicular to PQ, and equal to SQ; (I. 12) also SF parallel to RG, and HKN parallel to PQ. (I. 17)

Then KF is = the square on RS, for KN is = RS. \therefore the difference between the squares on RQ, RS is = the figure KQF.

Hence we have to show that the figure KQF is = the rectangle *PS*, *SQ*.

Now the rectangle QF is = the rectangle PK, for QS = PH, and QC = PR.

 \therefore the figure KQF is = the figure *PN*, which is = the rectangle *PS*, *SQ*, for *PH* is = *SQ*.

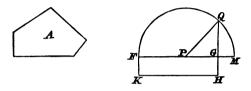
 \therefore the difference between the squares on RQ, RS is = the rectangle PS, SQ.

Wherefore the difference &c.

Q.E.D.

PROPOSITION VI.

To describe a square equal to a given rectilineal figure.



Let A be the given rectilineal figure. It is required to describe a square equal to A.

First describe the rectangle FGHK = A. (I. 24; II. 2) Produce FG, and cut off GM = GH.

Bisect FM in P, and with centre P and radius PM describe circle FQM.

Produce HG to meet the circumference in Q.

The square on GQ shall be = A.

Then since FG is = the sum and GM = the difference of PM and PG;

... the rectangle FG, GM is = the difference of the squares on PM and PG, (II. 5) and ... = the difference of the squares on PQ and PG,

and \therefore = the square on GQ; (I. 26) but *FH* is = the rectangle *FG*, *GM*, for *GH* = *GM*.

Therefore the square on GQ is = FH, and $\therefore = A$. Wherefore &c. Q.E.F.

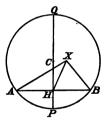
BOOK III.

CIRCLES.

THE CENTRE.

PROPOSITION I.

To find the centre of a given circle.



Let ABQ be the given circle. It is required to find the centre of the circle ABQ

Join A, B any two points in the circumference. Bisect AB in H. The Centre. 43

Through H draw a straight line perpendicular to AB, meeting the circumference in P, Q. (I. 12)

Bisect PQ in C. (I. 9)

Then shall C be the centre of the circle ABQ.

For if not, if possible, let some point without PQ, as X, be the centre, and join AX, HX, BX.

Then because AH, HX, XA are respectively = BH HX, XB,

 $\therefore \ \angle AHX \text{ is } = \ \angle BHX; \qquad (I. 7)$

 $\therefore \ \angle AHX$ is a right \angle ; but $\angle AHQ$ is a right \angle ;

 \therefore no point without PQ is the centre.

 \therefore the centre is in PQ, and is \therefore at its middle point C.

Wherefore the centre of the circle has been found.

Q. E. F.

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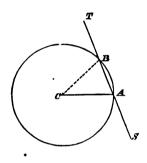
TANGENTS.

DEFINITION.

A straight line which meets a circle, and which, if produced, will not cut it, is said to **touch the circle**, and is called a **tangent** to it.

PROPOSITION III.

The straight line drawn through any point in the circumference of a circle perpendicular to the radius at that point touches the circle.



Let A be any point in the circumference of the circle AB. Join A with the centre C, and through A draw ST perpendicular to CA.

Then ST shall touch the circle.

For, if possible, let ST cut the circle in B. Join BC.

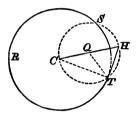
 \therefore SAT does not cut the circle in the point B.

Similarly it has no other point besides A in common with the circumference;

 \therefore SAT touches the circle at the point A. Wherefore the straight line &c. Q.E.D.

PROPOSITION IV.

To draw a tangent to a circle from a given point.



If the given point is on the circumference, then the straight line drawn perpendicular to the radius through that point will be a tangent. (III. 3)

If the given point is without the circle, as H, join it with C the centre of the given circle.

Bisect CH in Q; and with centre Q and radius QC or QH describe a circle cutting the given circle in T.

Join HT.

Then HT shall touch the given circle.

Join QT.

Tangents.

Because CQ = QT, $\therefore \angle QCT$ is $= \angle QTC$. (I. 5) So also $\angle QHT = \angle QTH$.

but the \angle s of the \triangle CTH are together = two right angles. (I. 20)

 $\therefore \ \angle CTH$ is a right angle.

 \therefore HT touches the circle RST. (III. 3)

Wherefore a tangent &c.

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Q.E.F.

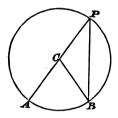
ANGLES IN CIRCLES.

DEFINITION.

An arc of a circle is a part of the circumference.

PROPOSITION V.

The angle at the centre of a circle is double of the angle at the circumference upon the same arc.



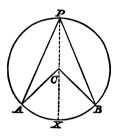
Let AB be an arc of a circle of which C is the centre, ACB the angle upon AB at the centre,

and *APB* an angle upon *AB* at the circumference. Then shall $\angle ACB$ be double of $\angle APB$.

ist. Let the centre lie in one of the arms AP or $\angle APB$.

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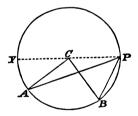
adly. Let C lie within the angle APB.



Join PC, and produce it to meet the circumference in X.

Then $\angle ACX$ is double of $\angle APX$, and $\angle BCX$ is double of $\angle BPX$; \therefore the whole $\angle ACB$ is double of $\angle APB$.

3dly. Let C lie without the $\angle APB$.



• Join PC, and produce it to meet the circumference in Y.

Then $\angle BCY$ is double of $\angle BPY$, and $\angle ACY$ is double of $\angle APY$; \therefore the remaining $\angle ACB$ is double of $\angle APB$. Wherefore the angle &c. Q.E.D.

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Primer of Geometry.

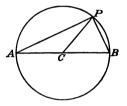
DEFINITION.

A segment of a circle is the figure contained by a part of the circumference and the straight line joining its extremities, which is called the **base** of the segment.

An angle in a segment is the angle contained by two straight lines drawn from any point in the arc of the segment to the extremities of the base.

PROPOSITION VI.

The angle in a semicircle is a right angle.



Let APB be an angle in the semicircle APB. Then shall $\angle APB$ be a right angle.

Join P with C, the centre of the circle of which the semicircle APB is a segment.

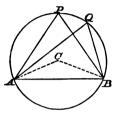
 $\therefore \ \angle APB$ is a right angle.

Wherefore the angle in a semicircle is a right angle. Q.E.D.

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PROPOSITION VII.

The angles in the same segment of a circle are equal to one another.



Let APB, AQB be angles in the same segment APQB. Then shall the \angle s APB, AQB be equal to one another.

First, let the segment APQB be greater than a semicircle.

Join A and B with C the centre of the circle of which APB is a segment.

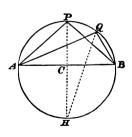
Then $\angle ACB$ is double of each of the $\angle s APB, AQB;$ (III. 5)

 \therefore \angle s *APB*, *AQB* are equal to one another.

Next, let the segment APQB be not greater than a semicircle.

Draw PCH through the centre C, and join HQ.

Then the \angle s *APH*, *AQH* are in a segment greater than a semicircle, and are \therefore equal to one another.

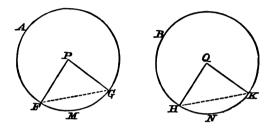


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Similarly \angle s *BPH*, *BQH* are equal to one another. \therefore the whole $\angle APB$ is = the whole $\angle AQB$. Wherefore the angles &c. Q.E.D.

PROPOSITION VIII.

In equal circles equal angles stand upon equal arcs.



Let AFM, BHN be equal circles, and let the \angle s FPG, HQK at their centres be equal to one another. Then shall the arc FMG be equal to the arc HNK.

For let the circle AFM be applied to the circle BHN, so that P may fall on Q,

and the circumference AFM on the circumference BHN;

then if PF fall on QH,

PG will fall on QK, since $\angle FPG$ is $= \angle HQK$ the arc FMG will fall on the arc HNK,

and is therefore equal to it.

Wherefore in equal circles &c. Q.E.D.

COROLLARY.—In equal circles, or in the same circle, equal chords cut off equal arcs.

For if the chord FG be = the chord HK;

since also PF and PG are respectively equal to QH and QK;

$$\therefore \ \angle FPG \text{ is } = \ \angle HQK; \qquad (I. 7)$$

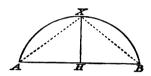
$$\therefore \text{ arc } FMG \text{ is } = \text{ arc } HNK.$$

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ON THE SUBDIVISION OF THE CIRCUMFERENCE OF A CIRCLE.

PROPOSITION IX.

To bisect a given arc of a circle.



Let AXB be the given arc of a circle.

It is required to bisect it.

Join AB and bisect it in H. (I. 9) Through H draw HX perpendicular to AB, (I. 12) cutting the arc in X. The arc AXB shall be bisected in X.

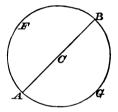
Join AX and XB.

Because AH = HB and HX is common to the Δ s AHX, BHX; and also $\angle AHX$ is $= \angle BHX$; $\therefore AX = BX$ (I. 4) \therefore arc AX is = arc BX (III. 8, Cor.)

Wherefore the given arc has been bisected. Q.E.F.

PROPOSITION X.

To bisect the circumference of a circle.



Let *AFB* be the given circle. It is required to bisect its circumference.

Through the centre C draw any diameter ACB, then shall the circumference be bisected by ACB.

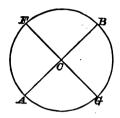
For if the circle were taken up, inverted, and then applied to its former position so that ACB should fall on its former position,

then would the arc AGB fall upon the arc AFB;

 \therefore arc $AGB = \operatorname{arc} AFB$.

Wherefore the circumference &c. Q.E.F.

COROLLARY.—Hence the circumference of a circle may be divided into 4, 8, 16, 32, &.c., equal arcs.



For if through C, FCG be drawn perpendicular to ACB, then FCG will bisect the arcs AGB, AFB; (III. 9)

.: the circumference will be divided into four equal parts.

Again, if the angles at the centre be bisected,

then each of the arcs AG, GB, BF, FA will be bisected, (III. 8)

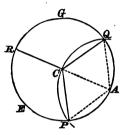
and \therefore the circumference will be divided into *eight* equal parts.

And so on.

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PROPOSITION XI.

To trisect the circumference of a circle.



Let *GEA* be the given circle.

It is required to trisect its circumference.

Join C the centre with any point A in the circumference.

With A as centre at distance AC describe a circle, cutting GEA in P and Q.

Produce AC to meet GEA in the point R.

Then shall the circumference of GEA be trisected in P, Q, R.

Join CP, CQ, AP, AQ.

Then Δs ACP, ACQ are equilateral, and \therefore equiangular; (I. 5)

 \therefore each of the \angle s ACP, ACQ is = a third of two right angles; (I. 20)

 \therefore each of the \angle s *PCR*, *QCR* is = two-thirds of two right angles: (I. 14)

also $\angle PCQ$ is = two-thirds of two right angles.

 \therefore \angle s PCR, QCR, PCQ are equal to one another.

 \therefore the arcs *PR*, *QR*, *PQ* are equal to one another. (III. 8)

Wherefore the circumference *GEA* has been trisected. Q.E.F.

COROLLARY.—Hence the circumference of a circle may be divided into 6, 12, 24, &.c., equal parts.

If PC, QC be produced to meet the given circumference GEA,

it will be divided into six equal arcs.

And so on.

BOOK IV.

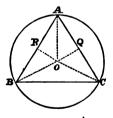
INSCRIBED AND CIRCUMSCRIBED FIGURES.

DEFINITION.

A circle is said to be *described about* a rectilineal figure when the circumference of the circle passes through all the angular points of the figure.

PROPOSITION I.

To describe a circle about a given triangle.



Let ABC be the given triangle. It is required to describe a circle about the $\triangle ABC$. Bisect AC in Q and AB in R.(I. 9)Through Q and R draw straight lines perpendicular toAC and AB intersecting in O.(I. 12)

Join OA, OB, OC.

With O as centre and OA as radius describe a circle; it shall be the one required.

Because AR, RO, and \angle ARO are respectively equal to BR, RO, and \angle BRO,

 $\therefore OA = OB.$ (I. 4) Similarly OA = OC;

 \therefore the circle passes through *B* and *C*, and \therefore is described about the \triangle *ABC*.

Wherefore a circle &c. Q.E.F.

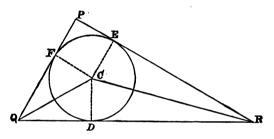
Primer of Geometry.

DEFINITION.

A circle is said to be **inscribed in** a rectilineal figure when each side of the figure touches the circle.

PROPOSITION II.

To inscribe a circle in a given triangle.



Let PQR be the given triangle; it is required to inscribe a circle within it.

Bisect \angle s PQR, PRQ by QC, RC, intersecting in C. (I. 8)

From C let fall CD, CE, CF perpendicular to QR, \cdot RP, PQ. (I. 13)

With C as centre and CD as radius describe a circle; it shall be the one required.

бо

Because the \angle s QDC, DQC are equal to the \angle s QFC, FQC,

also QC is common to the $\Delta s DQC$, FQC,

$$\therefore CD = CF.$$
 (I. 6)

Similarly CD = CE.

 \therefore the circle passes through F and E.

Again, because QR is perpendicular to CD,

 $\therefore QR$ touches the circle. (III. 3)

Similarly RP and PQ touch the circle.

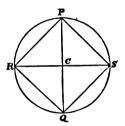
Wherefore a circle has been inscribed in the given triangle. Q.E.F.

DEFINITION.

A rectilineal figure is said to be **inscribed in a circle** when all its angular points are upon the circumference of the circle.

PROPOSITION III.

To inscribe a square in a given circle.



Let PRQ be the given circle; it is required to inscribe a square in it.

Draw the two diameters PCQ, RCS perpendicular to one another.

Join PS, SQ, QR, RP. Then PRQS shall be the square required.

Because PC, CR, and \angle PCR are respectively equal to QC, CR, and \angle QCR, \therefore PR = RQ. (I. 4) Similarly RQ = QS, and QS = SP.

... PROS is equilateral.

Again, since $\angle RPS$ is in a semicircle, \therefore it is a right angle. (III. 6)

Similarly the other \angle s of *PRQS* are right angles.

.: PRQS is a square.

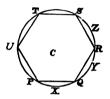
Wherefore a square has been inscribed in the given circle. Q.E.F.

DEFINITION.

A regular hexagon is a plane rectilineal figure bounded by six equal straight lines, and having its angles equal to one another.

PROPOSITION IV.

To inscribe a regular hexagon in a given circle.



Let PQR be the given circle; it is required to inscribe a regular hexagon in it.

Let the circumference be divided into six equal arcs *PXQ*, *QYR*, *RZS*, &c. (III. 11, Cor.) Draw the chords *PQ*, *QR*, *RS*, &c.

Then shall PORSTU be the regular hexagon required.

For if the figure were turned about C, so that P might coincide with the former position of Q,

and $\therefore Q$ with that of \hat{R} , R with that of S, and so on, since the arcs PXQ, QYR, &c., are all equal,

then the chord PQ would coincide with QR, and is $\therefore = it$:

similarly QR = QS, and so on.

... the hexagon is equilateral.

Also the arms of the $\angle PQR$ would fall where those of $\angle QRS$ were.

Hence $\angle PQR = \angle QRS$. Similarly $\angle QRS = \angle RST$; and so on.

... the hexagon is equiangular.

Wherefore a regular hexagon &c.

Q.E.F.

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